UNIT I

MATHEMATICAL TOOLS

1.1 Basic Mathematics for Physics

Mathematics is the TOOL of Physics. A good knowledge and applications of fundamentals of mathematics (which are used in physics) helps in understanding the physical phenomena and their applications. The topics introduced in this chapter enable us to understand topics of first year pre university physics.

I. Quadratic Equation and its Solution

A second degree equation is called **quadratic equation**.

The equation, $ax^2 + bx + c = 0$ is a quadratic equation,

In this equation, a, b and c are constants and x is a variable quantity.

The *solution* of the quadratic equation is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Illustration: Comparing the given quadratic equation $x^2 - 5x + 6 = 0$

with the standard form of quadratic equation $a\,x^2 + b\,x + c = 0$

We have $a = 1, b = -5, c = 6$

Now, we know $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\therefore x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times 6}}{2 \times 1}$

$$= \frac{5 \pm \sqrt{25 - 24}}{2} = \frac{5 \pm 1}{2}$$

$$x = \frac{6}{2} \text{ or } \frac{4}{2}$$

$$x = 3 \text{ or } x = 2$$

Exercise 1.1:
Solve for x comparing with the standard equation

1. $x^2 - 9x + 14 = 0$

2. $2x^2 + 5x - 12 = 0$

3. $3x^2 + 8x + 5 = 0$

4. $4x^2 - 4ax + (a^2 - b^2) = 0$

II. Binomial Theorem

According to this theorem, $(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-2)}{3!}x^3 + ...$

Where | x \ < 1, n is any negative integer or any fraction (positive or negative)

The total number of terms = $n + 1$ *i.e.* one more than the index of the power of the Binomial.
2! = 2 x 1, 3! = 3 x 2 x 1 and $n! = n(n-1)(n-2)(n-3)$..........1
If $|x| < <1$, then the terms containing higher power of x can be neglected.
Therefore **$(1+x)^n = 1 + nx$.**

Binomial theorem for positive integral index

$$(x+a)^n = x^n + \frac{nx^{n-1}a}{1!} + \frac{n(n-1)x^{n-1}}{2!}a^2 + ...+ a^n$$

where n is any positive integer.

Example 1: ***Expand*** **$(1+x)^{-2}$**

Solution: $(1+x)^{-2}$ $= 1 + (-2)x + \frac{(-2)(-2-1)}{2!}x^2 + \frac{(-2)(-2-1)(-2-2)}{3!}x^3 + ...$

$$= 1 - 2x + \frac{6}{2!}x^2 - \frac{24}{3!}x^3 + ...$$

$$= 1 - 2x + 3x^2 - 4x^3 + ...$$

Example 2: ***Evaluate*** $\sqrt{37}$ ***correct up to three decimal places.***

Solution: $\sqrt{37} = (36+1)^{1/2} = (36)^{1/2}\left(1+\frac{1}{36}\right)^{1/2} = 5(1+0.028)^{1/2}$

$$= 5\left[1 + \frac{1}{2}(0.028) + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}(0.028)^2 + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(0.028)^3 + ...\right]$$

We have neglected the terms containing powers of 0.028.

$$\therefore \sqrt{37} = 6[1 + 0.014]$$

$$= 6[1 + 0.014] = 6[1.014]$$
$$= 6.084$$

Exercise:

1. The value of acceleration due to gravity (g) at a height h above the surface of earth is

$g' = \frac{gR^2}{(R+h)^2}$ ***If h<<R, then*** **prove that** $\boldsymbol{g' = g\left(\frac{1 - 2h}{R}\right)}$

Hint: $g' = \left(\dfrac{gR^2}{R^2\left(1+\dfrac{h}{R}\right)^2} \right) = g\left(1+\dfrac{h}{R}\right)^{-2}$

2. ***Solve (1 + x)^3 using Binomial theorem.***

III. Logarithms

If $a^x = m$, then x is called the logarithm of m to the base a and is written as $\log_a m$ Thus, if $a^x = m$, then $\log_a m = x$

For example (i) If $2^4 = 16 \therefore \log_2 16=4$ (ii) $3^3 = 27 \therefore \log_3 27=3$

(iii)$\log_a 1=0$ (iv) $\log_a a = 1$.

Standard Formulae of logarithms

1. $\log_e mn = \log_e m + \log_e n$ 2. $\log_e \dfrac{m}{n} = \log_e m - \log_e n$ 3. $\log_e m^n = n \log_e m$

Two Systems of Logarithms

1. **Natural Logarithm.** *Logarithm of a number to the base e (e = 2.7182) is called natural logarithm.*
2. **Common Logarithm.** *Logarithm of a number to the base 10 is called common logarithm.* In all practical calculations, we always use common logarithm.

Conversion of Natural logarithm to Common logarithm

Natural logarithms can be converted into common logarithms as follows:

$\log_e N = 2.3026 \log_{10} N$

$\cong 2.303 \log_{10} N$

Example:

Work done during an isothermal process is $W = RT \log_e \dfrac{V_2}{V_1}$

This can be written as W = 2.3026 $RT \log_{10} \dfrac{V_2}{V_1} \approx 2.0303 RT \log_{10} \dfrac{V_2}{V_1}$

Example: ***Expand the following using logarithm formulae***

(i) $f = \frac{1}{2l}\sqrt{\frac{T}{m}}$

Solution:

$$f = \frac{1}{2l}\left(\sqrt{\frac{T}{m}}\right)^{1/2}$$

Taking log both sides, we get

log f = log $T^{1/2}$ – log $m^{1/2}$ – (log 2 + log *l*)

$$= \frac{1}{2}\log T - \frac{1}{2}\log m - (\log 2 + \log l) = \frac{1}{2}(\log T - \log m) - \log 2 - \log l.$$

Exercise

Expand the following by using logarithm formulae

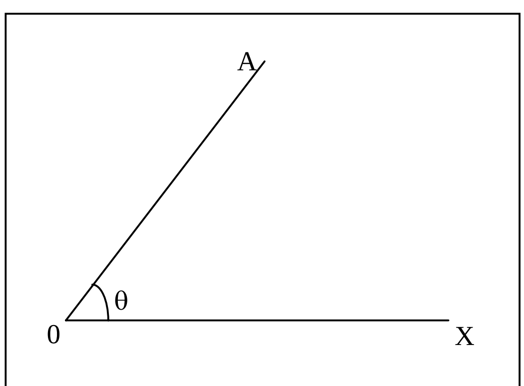
(i) $PV^{\gamma} = K$ (ii) $V = \frac{\pi \Pr^4}{8\eta l}$ (iii) $h = \frac{2T}{rpg}$ (iv) $T = 2\pi\sqrt{\frac{l}{g}}$

IV. Trigonometry

Angle: Consider a fixed straight line OX. Let another straight line OA (called revolving line) be coinciding with OX rotate anticlockwise and takes the position OA, The angle is measured by the amount of revolution that the revolving line OA undergoes in passing from its initial position to final position.

From Figure given below, angle covered by revolving line OA is

θ = ∠AOX.



An angle ∠AOX is +ve, if it is traced out in anticlockwise direction and ∠AOX is –ve, if it is traced out in clockwise direction

System of Measurement of an Angle

(i) Sexagesimal System (ii) circular system

(i) Sexagesimal System:

In this system

1 right angle = 90^{O} (degrees)

1 degree = $60'$ (minutes)

1 minute = $60''$ (seconds)

(ii) In circular system :

π **Radians = 180^{O} = 2 right angles**

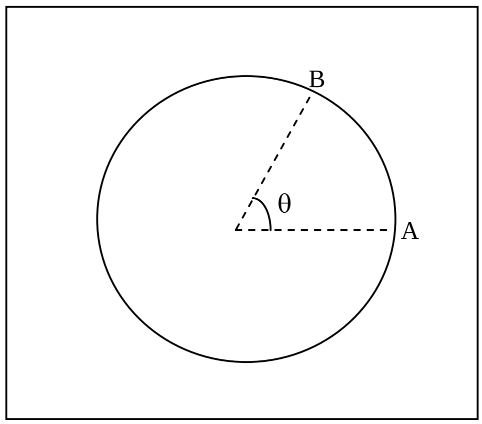
$\therefore$ **1right. Angle = $\frac{\pi}{2}$ radians.**

Let a particle moves from initial position A to the final position B along a circle of radius *r* as shown in figure.

Then, Angle, $\theta = \dfrac{Lenght\ of\ arc\ AB}{Radius\ of\ circle\,(r)}$

If length of arc AB = radius of the circle (*r*)

Then θ = 1 radian



Radian: *An angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle is called one radian.*

Relation between Radian and Degree

When a body or a particle completes one rotation, then θ = 360° and distance travelled (circumference of a circle).

$$\theta = \frac{2\pi r}{r}$$

Or $360^{O} = 2\pi$ radian

Or 1 rad = $\frac{360}{2\pi} = \frac{180}{22} \times 7$

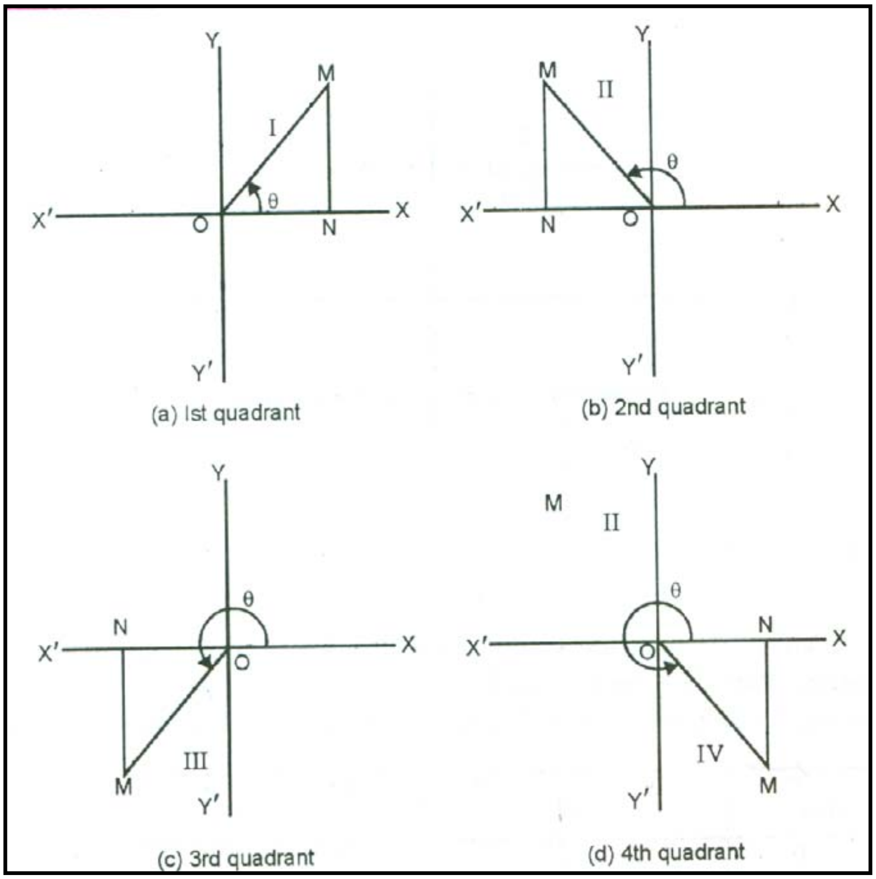
$$= 57.27^{O}$$

Thus, 1 radian = 57.27^{O}

Trigonometric Ratios

Consider triangle ONM in the four quadrants as shown below.

Consider two straight lines X'OX and Y'OY meeting at right angles in O. These two lines divide the plane into four equal parts called *quadrants* (figure given below).



Now XOY, YOX', X'OY' and Y'OX are called I, II, III, and IV *quadrants* respectively. ON is +ve if drawn to the right side of O and –ve if drawn to the left side of O. MN is +ve if drawn above X'OX and –ve if drawn below X'OX,

Trigonometric Ratios of an Angle

1. $\frac{MN}{OM}$ = sin θ (i.e. sine of θ)
2. $\frac{ON}{OM}$ = cos θ (i.e. cosine of θ)
3. $\frac{MN}{ON}$ = tan θ (i.e. tangent of θ)
4. $\frac{OM}{MN}$ = cosec θ (i.e. cosecant of θ)
5. $\frac{OM}{ON}$ = sec θ (i.e. secant of θ)
6. $\frac{OM}{MN}$ = cos θ (i.e. cotangent of θ)

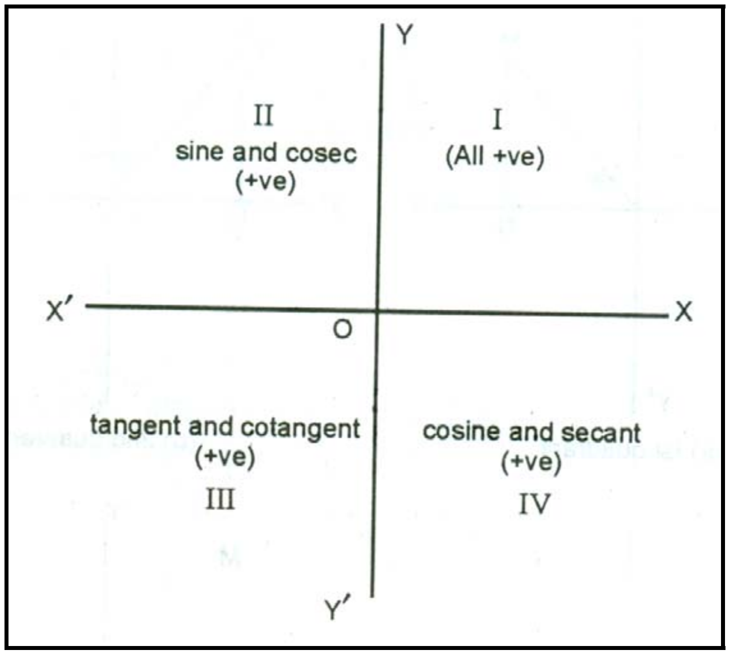
These ratios are called trigonometric ratios.

Important relations:

1. cosec θ = $\frac{1}{\sin\theta}$ 2. sec θ = $\frac{1}{\cos\theta}$ 3. cot θ = $\frac{1}{\tan\theta}$

4. $\sin^2\theta + \cos^2\theta = 1$ 5. $\sec^2\theta = 1 + \tan^2\theta$ 6. $\text{cosec}^2\theta = 1 + \cot^2\theta$

Signs of trigonometric ratios



The signs of various trigonometric ratios can be remembered from the above figure.

Trigonometric Ratios of Standard angles

The trigonometric ratios of standard angles are given in the following table:

Angle θ → **trig-ratio ↓**	**0^O**	**30^O**	**45^O**	**60^O**	**90^O**	**120^O**	**180^O**
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	0
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	–1
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	0

Trigonometrical Ratios of Allied Angles

1. (i) $\sin(-\theta) = -\sin\theta$ (ii) $\cos(-\theta) = \cos\theta$ (iii) $\tan(-\theta) = -\tan\theta$
2. (i) $\sin(90^O-\theta) = \cos\theta$ (ii) $\cos(90^O-\theta) = \sin\theta$ (iii) $\tan(90^O-\theta) = \cot\theta$
3. (i) $\sin(90^O+\theta) = \cos\theta$ (ii) $\cos(90^O+\theta) = \sin\theta$ (iii) $\tan(90^O+\theta) = -\cot\theta$
4. (i) $\sin(180^O-\theta) = \sin\theta$ (ii) $\cos(180^O-\theta) = -\cos\theta$ (iii) $\tan(180^O-\theta) = -\tan\theta$
5. (i) $\sin(180^O+\theta) = -\sin\theta$ (ii) $\cos(180^O+\theta) = -\cos\theta$ (iii) $\tan(180^O+\theta) = -\tan\theta$
6. (i) $\sin(270^O-\theta) = -\cos\theta$ (ii) $\cos(270^O-\theta) = -\sin\theta$ (iii) $\tan(270^O-\theta) = \cot\theta$
7. (i) $\sin(270^O+\theta) = -\cos\theta$ (ii) $\cos(270^O+\theta) = \sin\theta$ (iii) $\tan(270^O+\theta) = -\cot\theta$

Illustrations:

Find the values of

(i) $\sin 270^O$ (ii) $\sin 120^O$ (iii) $\sin 120^O$ (iv) $\tan(-30^O)$

Solution:

(i) $\sin 270^O = \sin(180^O + 90^O) = -\sin 90^O = -1$

(ii) $\cos 120^O = \cos(90^O + 30^O) = -\sin 30^O = -\frac{1}{2}$

(iii) $\sin 120^O = \sin(90^O + 30^O) = \cos 30^O = \frac{\sqrt{3}}{2}$

(iv) $\tan(-30^O) = -\tan 30^O = -\frac{1}{\sqrt{3}}$

Some important Trigonometric Formulae

1. $\sin(A+B) = \sin A\cos B + \cos A\sin B$
2. $\cos(A+B) = \cos A\cos B - \sin A\sin B$
3. $\sin(A-B) = \sin A\cos B - \cos A\sin B$
4. $\cos(A-B) = \cos A\cos B + \sin A\sin B$
5. $\sin 2A = 2\sin A\cos A$
6. $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$
7. $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$
8. $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$
9. $\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
10. $\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
11. $\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
12. $\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$

13. $\cos 2A = 1 - 2\sin^2 A = 2\cos^2 A - 1$

14. $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

15. $\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A \tan B}$

16. $\tan(A+B) = \dfrac{\tan A + \tan B}{1-\tan A \tan B}$

V. Differentiation

Function: If the value of a quantity *y* (say) depends on the value of another quantity *x*, then *y* is the function of *x i.e.* $y = f(x)$.

The quantity y is called dependent variable and the quantity x is called independent variable. For example, $y = 2x^2 + 4x + 7$ is a function of *x*

(i) When $x = 1$, $y = 2(1)^2 + 4\text{x}1+7 = 13$

(ii) When $x = 2$, $y = 2(2)^2 + 4\text{x}2+7 = 23$

As the value of *y* depends on the value of *x*, *y* is the function of *x*.

Differential coefficient or derivative of a function

Let $y = f(x)$ …. (1)

That is, the value of *y* depends upon the value of *x*.

Let Δx be a small increment in x, so that Δy is the corresponding small increment in y, then

$$y + \Delta y = f(x+\Delta x) \qquad \text{…. (2)}$$

Subtract (1) from (2), we get $\Delta y = f(x+\Delta x) - f(x)$

Divide both sides by Δx

$$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Where $\dfrac{\Delta y}{\Delta x}$ is called average rate of change of y w.r.t. *x*.

Let us Δx be as small as possible i.e. $\Delta x \to 0$ (read as delta x tends to zero)

Then differential coefficient or derivative of *y* w.r.t. *x* is

$$\therefore \frac{dy}{dx} = \underset{\Delta x \to 0}{Lt} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Theorems of Differentiations

1. If $y = C$, when C is constant $\therefore \dfrac{dy}{dx} = 0$
2. If $y = x^n$, where n is an integer $\therefore \dfrac{dy}{dx} = nx^{n-1}$

3. If $y = Cu$, where u is the function of x and C is constant $\frac{dy}{dx} = C\frac{du}{dx}$

4. If y = u ± v ± ω ± ……, where u, v and ω are the function of x

   $\therefore \frac{dy}{dx} = \frac{d}{dx}(u) \pm \frac{d}{dx}(v) \pm \frac{d}{dx}(\omega) \pm ....$

5. If $y = u\,v$, where u and v are the function of x, then $\frac{dy}{dx} = u\frac{dv}{dx} + \frac{du}{dx}$

6. If $y = \frac{v}{u}$, where u and v are the function of x, then $\frac{dy}{dx} = \frac{u\frac{dv}{dx} - v\frac{du}{dx}}{u^2}$

7. If $y = u^n$, where u is the function of x then $\frac{dy}{dx} = nu^{n-1}\frac{du}{dx}$

Exercise 1.2

1, Find derivative of the functions w.r.t x

(i) $4x^3+7x^2+6x+9$ **(ii)** $\frac{x^5}{2} - \frac{5}{x^2}$ **(iii)** $\frac{1}{\sqrt{x}}$ **(iv)** $\frac{1}{x+4}$

Differential coefficients of Trigonometric Functions

1. $\frac{d}{dx}(\sin x) = \cos x$; and $\frac{d}{dx}(\sin u) = \cos u\,\frac{d(u)}{dx}$, u is the function of x

2. $\frac{d}{dx}(\cos x) = -$ six x ; $\frac{d}{dx}(\cos u) = -\sin u\,\frac{d}{dx}(u)$

3. $\frac{d}{dx}(\tan x) = \sec^2 x$; $\frac{d}{dx}(\tan u) = \sec^2 u\,\frac{d}{dx}(u)$

4. $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

5. $\frac{d}{dx}(\sec x) = \sec x \tan x$

6. $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x.$

Example:

Differentiate the following w.r.t. *x*.

(i) sin 2 *x* **(ii) *x* sin *x***

Solution:

(i) Let $y = \sin 2x$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(\sin 2x) = \cos 2x \frac{d}{dx}(2x) = \cos 2x.2 = 2\cos 2x$$

(ii) Let y = x sin x

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(x\sin x) = x\frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}(x) = x\cos x + \sin x.1$$

$$\therefore \frac{dy}{dx} = x\cos x + \sin x$$

Exercise 1.3

(i) sin 3*x* (ii) cos 3*x*

(iii) tan 4*x* (iv) sin (*ax* + *b*)

(v) cos *(ax + b)*

Differential Coefficients of Logarithmic and Exponential Functions

1. $\frac{d}{dx}(\log_e x) = \frac{1}{x}\log_e e = \frac{1}{x}\ ((\because \log_e e = 1)$

2. $\frac{d}{dx}(\log_e u) = \frac{1}{u}\frac{d}{dx}(u)$

3. $\frac{d}{dx}(e^u) = e^u \frac{d}{dx}(u)$

4. $\frac{d}{dx}(e^x) = e^x \log_e e = e^x$

5. $\frac{d}{dx}(e^u) = e^u \frac{d}{dx}(u)$

Example:

1. Differentiate the follow *w.r.t .x.*

(i) $(\log_e x)^2$ **(ii)** $\log(ax+b)$

Solution:

(i) Let y = $(\log_e x)^2$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(\log x)^2 = 2\log x \frac{d}{dx}(\log x) = 2\log x.\frac{1}{x} = \frac{2}{x}\log x$$

(ii) Let $y = \log(ax+b)$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\log(ax+b) = \frac{1}{(ax+b)}\frac{d}{dx}(ax+b) = \frac{a}{ax+b}$$

Example 3: ***If*** $S = 2t^3 - 3t^2 + 2,$ ***find the position, velocity and acceleration of a particle at the end of 2s. S is measured in metre and t in second.***

Solution:

$S = 2t^3 - 3t^2 + 2,$

When t = 2s, S = 2 × 8 – 3 × 4 + 2 = 6 m

Now, velocity $v = \frac{dS}{dt} = \frac{d}{dt}(2t^3 - 3t^2 + 2) = 6t^2 - 6t$

When $t - 2s, v = 6\times4 - 6\times2 = 12ms^{-1}$

Now, acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}(6r^2 - 6t) = 12t - 6$

When t = 2s, $a = 12\times2 - 6 = 18ms^{-2}$

Exercise 1.4

1. The area of a blot of ink is growing such that after t second $A = 3r^2 + \frac{t}{5} + 7$ Calculate the rate of increase of area after 5 s.
2. If the motion of a particle is represented by $S = t^3 + t^2 - t + 2$, find the position, velocity and acceleration of the particle after 2 s.
3. A particle starts rotating from rest and its angular displacement is given by $\theta = \frac{t^2}{40} + \frac{t}{5}$ Calculate the angular velocity at the end of 10 s.

VI. Integration

Integration is an inverse process of differentiation.

It is the process of finding the function whose derivative is given.

Suppose *F(x)* is the derivative of the function *f(x)* w.r.t.x. Then we can write

$$\frac{d}{dx} f(x) = F(x)$$

Now, if we are given the derivative *F(x)* and we have to find the function *f(x)* then this can be done with the help of Integral Calculus.

The process of finding the function whose derivative is given is called integration.

Definition: If the derivative of a function *f(x)* is *F(x)* then *f(x)* is called the integral of *F(x)* with respect to *x*. The integration of a function can be written as $\int F(x)dx = f(x)$
The function *F(x)* whose integral is *f(x)* is called Integrand.

Fundamental Formulae of Integration

1. $\int dx = x \left[\because \frac{d}{dx}(x) = 1\right]$

2. $\int x^n \; dx = \frac{x^{n+1}}{n+1}$ provided n $\neq -1$.

3. $\int x^{-1} dx = \int \frac{1}{x} dx = \log_e x$

4. $\int e^x dx = e^x$

5. $\int e^{ax} dx = \frac{e^{ax}}{\frac{d}{dx}(ax)} = \frac{e^{ax}}{a}$

6. $\int a^x dx = \frac{a^x}{\log_e a}$

7. $\int \sin x dx = -\cos x$

7a. $\int \sin ax dx = \frac{-\cos ax}{\frac{d}{dx}(ax)} = \frac{-\cos ax}{a}$

8. $\int \cos x dx = \sin x$

8a. $\int \cos ax dx = \frac{\sin ax}{a}$

9. $\int \sec^2 x dx = \tan x$

10. $\int \cos ec^2 x \; dx = -\cot x$

11. $\int \sec x \tan x \; dx = \sec x$

12. $\int \cos ecx \cot x \, dx = -\cos ecx$

13. $\int \tan x \; dx = \int \frac{\sin x}{\cos x} dx = -\int \frac{-\sin x}{\cos x} dx = -\log_e \cos x$

14. $\int \cot x \; dx = \int \frac{\cos x \; dx}{\sin x} = \log_e \sin x$

15. $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x; |x| < 1$

16. $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1}|x|;\; |x| > 1$

17. $\int \frac{dx}{1+x^2} = \tan^{-1} x$

Theorems of Integration

First Theorem: The integral of the product of *a constant* and a function is equal to the product of the constant and integral of the function.

i.e. $\int c\, u\, d\, x = c\int u\, d\, x$, where c is constant.

Second Theorem: The integral of the *sum* or difference of a number of functions is given by

$$\int (u \pm v \pm \omega \pm ....)\; dx = \int u dx \pm \int v\, d\, x \pm \int \omega\, d\, x \pm ....$$

Exercise 1.5

Integrate the following functions w.r.t .x,

(i) x^3 **(ii)** $x^2 + \frac{1}{x}$ **(iii)** e^{3x} **(iv)** $\left(x - \frac{1}{x}\right)^2$ **(v)** $\frac{1}{\sqrt{x}}$ **(vi)** $4e^{5x}$

Definite integral

If

$$\int F(x)dx = f(x) + c$$

Then $\int_a^b F(x)dx = f(b) - f(a)$

Where a and b are called the upper and lower limits of x

Definite integral is illustrated with the following examples.

Exercise 1.6

Solve the following:

(i) $\int_{x=\infty}^{x=R} \frac{GMm}{x^2} dx$ **(ii)** $\int_0^{\pi/2} \cos x \; dx$ **(iii)** $\int_1^2 x^3 \; dx$

(iv) $\int_u^v m v \, d v$ **(v)** $\int_{-\pi/2}^{\pi/2} \sin x \; d x$

Exercise 1.7

Solve the following:

(i) $\int_{-\pi/2}^{\pi/2} \cos x \, dx$ **(ii)** $\int_0^Q \frac{q}{C} dq,$ ***where C is a constant***

(iii) $\int_{\theta_1}^{\theta} \frac{d\theta}{(\theta - \theta_0)'}$ ***where, θ_0 is a constant***

MATHEMATICAL TOOLS

1.1 Basic Mathematics for Physics

Answers

Exercise 1.1:

1. (2, 7); 2. $\left(\frac{3}{2},-4\right)$; (3) $\left(-1,-\frac{5}{3}\right)$; (4) $\left(\frac{a+b}{2},\frac{a-b}{2}\right)$;

Exercise 1.2

(i) $12x^2 + 14x + 6$ (ii) $(5/2)\, x^4 - 5/\, 2x^3$ (iii) $\frac{1}{2x^{3/2}}$ (iv) $-\frac{1}{(x+4)^2}$

Exercise 1.3

(i) 3 cos 3x (ii) – 3 sin 3x (iii) 4 $\sec^2 4x$

(iv) a cos $(ax + b)$

Exercise 1.4

1. 30.2 2. 12, 15, 10 3. 0.7.

Exercise 1.5

Answers.

(i) $\frac{x^4}{4}$ (ii) $\frac{x^3}{3}+\log_e x$ (iii) $\frac{e^{3x}}{3}$

(iv) $\frac{x^3}{3}-2x-\frac{1}{x}$ (v) $2\sqrt{x}$ (vi) $\frac{4}{5}e^{5x}$

Exercise 1.6

Solution:

(i) $$\int_{\infty}^{R}\frac{GMm}{x^2}dx = GMm\int_{\infty}^{R}\frac{1}{x^2}dx = GMm\int_{\infty}^{R}x^{-2}dx = GMm\left[\frac{x^{-1}}{-1}\right]_{\infty}^{R}$$

$$= -GMm\left[\frac{1}{x}\right]_{\infty}^{R} = -GMm\left[\frac{1}{R}-\frac{1}{\infty}\right]$$

$$= -\frac{GMm}{R} because \left[\frac{1}{\infty}=0\right]$$

(ii) $\int_0^{\pi/2} \cos x \ dx = [\sin x]_0^{\pi/.2} = \left[\sin\frac{\pi}{2} - \sin 0\right] = 1 - 0$

(iii) $\int\limits_1^2 x^3 dx = \left[\frac{x^4}{4}\right]_1^2 = \frac{(2)^4}{4} - \frac{(1)^4}{4} = \frac{16}{4} - \frac{1}{4}$

$= \frac{15}{4}$

(iv) $\int\limits_u^v m\, v\, d\, v = m \int\limits_u^v v\, d\, v = m\left[\frac{v^2}{2}\right]_u^v = m\left[\frac{v^2}{2} - \frac{u^2}{2}\right]$

$= \frac{m}{2}(v^2 - u^2)$

(v) $\int\limits_{-\pi/2}^{\pi/2} \sin x\, dx = -[\cos x]_{-\pi/2}^{\pi/2} = -\left[\cos\frac{\pi}{2} - \cos(-\pi/2)\right] = 0$

Exercise 1.7

(i) 2 (ii) $\frac{Q^2}{2C}$ (iii) $\log\frac{\theta_2 - \theta_0}{\theta_1 - \theta_0}$